- APPENDIX -

MAYARD'S PROBLEM SOLVING METHOD

4 simple steps

U: Understand the problem and the question to be resolved

I: Identify the variables and the data provided in the question.

S : Start to solve with what you know

V : Verify your answer

Note: A sample of word problems from high school (2nd cycle) to college level is used to illustrate this simple method that can also be used in all grade levels.

Math– Grade 9

A cylindrical can contains 3 tennis balls with a radius of 4 cm as illustrated bellow:

. What is the volume of the unoccupied space in the cylinder ?

Solution:

1. U: Understand the problem and the question

We want to find the volume of the unoccupied space (Vu)

2. I: Identify the variable and the data

$r_{tennis} = r_{sphere} = 4$ cm; $r_{cylinder} = 4$ cm ; $h_{cylinder} = ?$

3.S: start to solve with what you know

***Remark**: Starting with what you know will enable you to see the other steps that must be completed.*

● The unoccupied volume in the cylinder is given by

the following operation :

$V_u = V_{cylinder} - 3V_{sphere}$ $\Rightarrow$ *Next step: find $V_{cylinder}$ and V_{sphere}*

● The volume of the cylinder is given by the following formula:

$$V_{cylinder} = A_b \times h_{cylinder}$$

$$V_{cylinder} = \pi 4^2 \times (4 \times 2 \times 3)$$ ($h_{cylinder}$ = 3 times the diameter of the tennis ball)

$$V_{cylinder} = 384\pi cm^3$$

● The volume of 1 sphere is given by the following formula:

$$V_{sphere} = \frac{4\pi r^3}{3}$$

$$V_{sphere} = \frac{4\pi \times 4^3}{3}$$

$$V_{sphere} = \frac{256\pi}{3} cm^3$$

● The unoccupied volume in the cylinder is:

$$V_u = V_{cylinder} - 3V_{sphere}$$

$$V_u = 384\pi - \frac{3 \times 256\pi}{3}$$

$$V_u = 128\pi \approx 402.12 cm^3$$

4. Verification

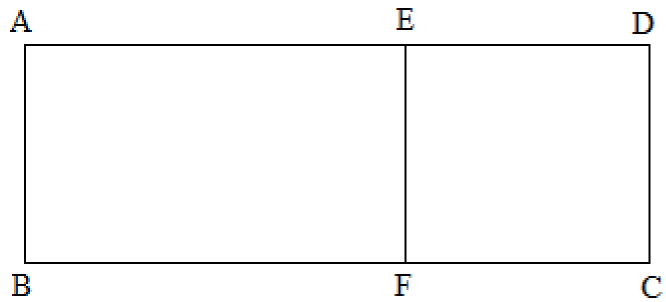
The answer makes sense if the unoccupied volume (Vu) is smaller then the volume of the cylinder ($V_{cylinder}$) and the volume of the 3 spheres ($3V_{sphere}$). Let's verify?

$V_u = 128\pi cm^3 < V_{cylinder} = 384\pi cm^3$ and $V_u = 128\pi cm^3 < 3V_{sphere} = 256\pi cm^3$ **(yes)**

Scientific Math – Grade 10 (Multiple steps problem)

Consider the rectangle ABFE and the square CDEF below. The segment AE has 2 units more than segment ED .

If the area of rectangle ABCD is $40\,cm^2$, what is the numerical value of the area of the rectangle ABFE.



Solution

1. **U : understand the problem and the question to solve.**

Problem : this situation involves algebraic concepts

Question to be resolved: Find the area (quantitative value) of rectangle ABFE

2. **I : Identify the variables and the important data**

Segment AE : 2 units more than segment ED

Area of rectangle ABCD is : $40\ cm^2$

3. **S : start with what you know**

Remark: Starting with what you know will enable you to see the other steps that must be completed.

● Area of rectangle ABCD :

Aire ABCD = Length x width

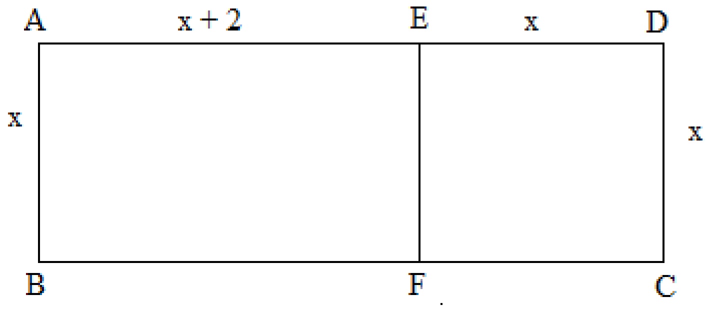
$\Rightarrow$ therefore we need to find the length (L) and the width (W):

if ED = x $\Rightarrow$ AE = x + 2

$\Rightarrow$ Length of AD = AE + ED = (x +2) + x = 2x + 2

$\Rightarrow$ Width AB = x (Since CDEF is a square)

● Now we can find the area:



Area of rectangle ABCD = $(2x+2)\bullet x = 2x^2 + 2x$

$40 = 2x^2 + 2x$

$0 = 2x^2 + 2x - 40$

Now we use the quadratic formula:

a = 2 b = 2 c = -40

$\Delta = b^2 - 4ac$

= 4- 4(2) (-40)

= 324

$$\Rightarrow \begin{array}{l} x = \dfrac{-b \pm \sqrt{\Delta}}{2a} \\ x = \dfrac{-2 \pm \sqrt{324}}{2(2)} \end{array}$$

$x = 4$ Or x = -5 (rejected)

$\Rightarrow$ AE = x + 2 = 4+2 = 6 cm

Area of rectangle ABFE :

Area of ABFE = x (x+2) = 4 (6) = $24cm^2$

4.Verify your answer

● Does segment AE have 2 more units than segment ED?

ED = 4 cm and AE = 6 cm **(yes)**

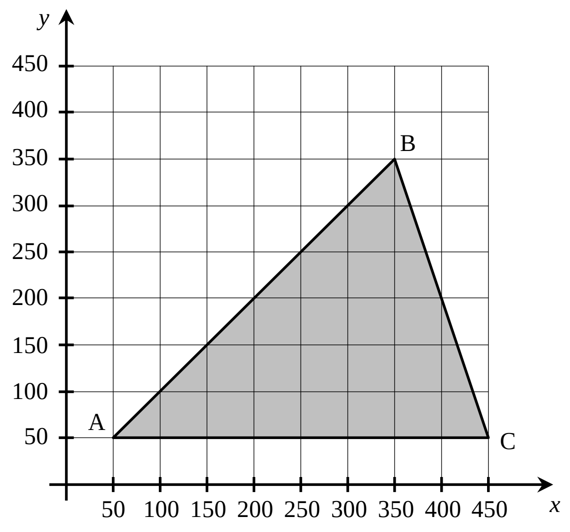
● Is the area of rectangle ABCD 40 cm^2 ?

ABCD = $(2x+2)\bullet x$

ABCD = [2(4) + 2]●4 = 40 cm^2 **(yes)**

Scientific /Cultural Math – Grade 11

A large boat transports people and motorcycles. Polygon of constraints ABC below is based on the constraints the transport company must take into account in order to make each crossing profitable.



Coordinates of the vertices of the polygon of constraints
A(50, 50) B(350, 350) C(450, 50)

x: number of people aboard for each crossing
y: number of motorcycles aboard for each crossing

The fare is \$4 per person and \$10 per motorcycle.

To ensure profitability, the company must take into account a new constraint represented by the rule $x + y \geq 300$.

By how much does this new constraint increase the minimum revenue for each crossing?

Solution:

1. U: Understand the problem and the question

Problem : this situation involves the notion of optimization
Question to be resolved: Find the increase in the minimum revenue that is the difference between the minimum revenue **with the new constraint** and **without it.**

2. I: Identify the variable and the data

x: number of people aboard for each crossing
y: number of motorcycles aboard for each crossing
Fare is \$4 per person and \$10 per motorcycle ⇒ Function rule: $P = 4x + 10y$
Added constraint: $x + y \geq 300$

3.S: start to solve with what you know

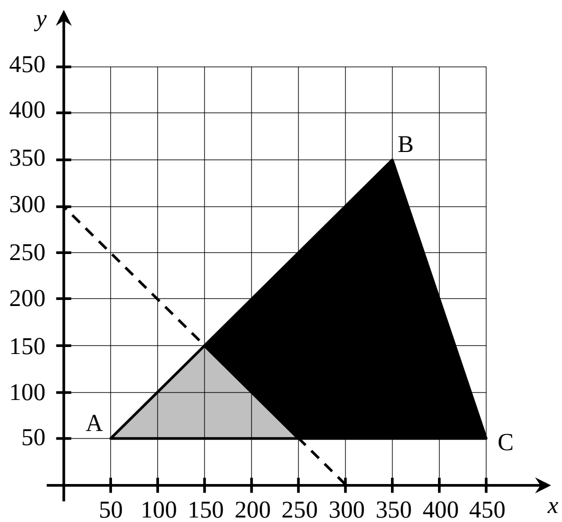
***Remark**: Starting with what you know will enable you to see the other steps that must be completed.*

● The increase in the minimum revenue (M) is given by the following operation:
$M_{\text{increase}} = M_{\text{with new constraint}} - M_{\text{without new constraint}}$ ⇒Next steps: Find the minimum revenue without the new constraint and with the new constraint.

● The minimum revenue **without** the new constraint is found with the function rule:

Vertex	Revenue: $4x + 10y$	
A(50, 50)	\$700	← Minimum
B(350, 350)	\$4900	
C(450, 50)	\$2300	

● The minimum revenue **with** the new constraint is found with the graph and function rule:
New constraint : $x + y \geq 300$



The coordinates of the 2 new vertices are :(150, 150) and (250, 50).

We use the function rule to find the minimum revenue with the new constraint:

Vertices	Revenue: $4x + 10y$	
(150, 150)	\$2100	
B(350, 350)	\$4900	
C(450, 50)	\$2300	
(250, 50)	\$1500	← Minimum

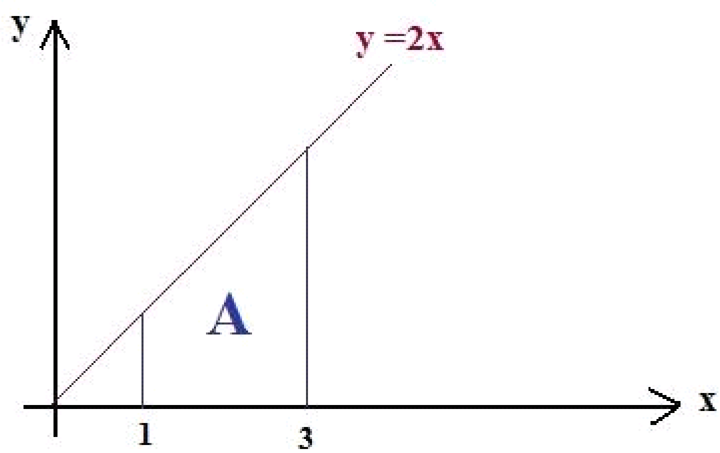
● The increase in the minimum revenue $M_{increase}$ is :
$M_{increase} = M_{with\ new\ constraint} - M_{without\ new\ constraint}$
$M_{increase} = \$1500 - \$700$
$M_{increase} = \$800$

4.Verification

The answer makes sense if the minimum revenue with the new constraint is greater than the minimum revenue without the new constraint. Let's verify?
$M_{with\ new\ constraint} = \$1500 > M_{without\ new\ constraint} = \$700$ **(yes)**

Calculus II – College

Given the graph below:



Find the area of the geometric figure bounded by the function y = 2x and the equations x =1 and x =3.

Solution

1.U: Understand the problem and the question

Problem: This situation involves the concept of definite integral and the notion of area under the curve.
Question to be resolved: We want to find the area under the linear function which has specific interval

2. I: Identify the variable and the data

function: y =2x ; slices along x: dx ; interval: [1, 3]

3.S: start to solve with what you know

● The area (A) is given by the definite integral :

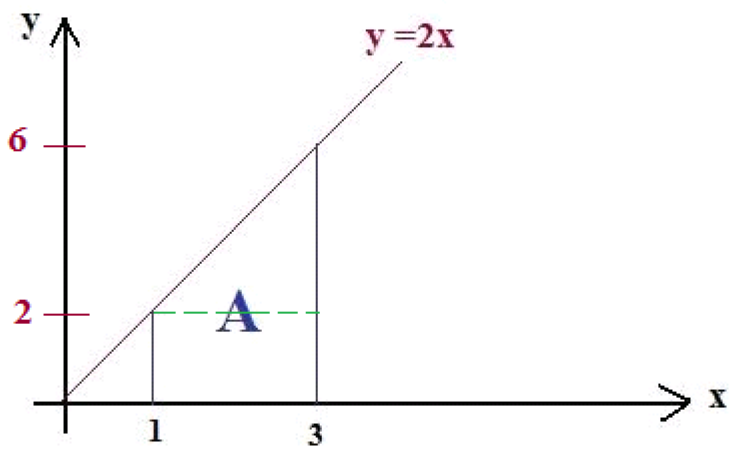
$$A = \int_1^3 2x \; dx$$

$$A = \Big|_1^3 (x^2) \qquad \text{(By rule of integration)}$$

$$A = (3^2) - (1^2) = 8u^2$$

4. Verification

● The area found with integration should also be equal to the sum of the area of the square and the area of the triangle.



Let's verify:

$$A_{integration} = A_{square} + A_{triangle}$$

$$A_{integration} = (2 \times 2) + (\frac{2 \times 4}{2})$$

$$A_{integration} = 4 + 4 = 8u^2$$ **(yes)**